

For a Discrete Time Markov chain defined by the following transition matrix P, identify all state classes.

$$P = \begin{bmatrix} \star & 0 & 0 & \star & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \star & 0 & 0 & 0 & \star & 0 & 0 & 0 & 0 \\ 0 & 0 & \star & 0 & 0 & 0 & 0 & 0 & \star & \star \\ \star & 0 & 0 & \star & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \star & \star & \star & 0 & \star & \star & 0 & 0 & 0 \\ 0 & \star & 0 & 0 & 0 & \star & 0 & 0 & 0 & 0 \\ 0 & \star & \star & 0 & \star & 0 & \star & \star & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \star & 0 \\ 0 & 0 & \star & 0 & 0 & 0 & 0 & 0 & 0 & \star \\ 0 & 0 & \star & 0 & 0 & 0 & 0 & 0 & \star & \star \end{bmatrix}$$

I have 4 umbrellas, some at home, some in the office. I keep moving between home and office. I take an umbrella with me only if it rains. If it does not rain I leave the umbrella behind (at home or in the office). It may happen that all umbrellas are in one place, I am at the other, it starts raining and must leave, so I get wet.

1. If the probability of rain is  $p$ , what is the probability that I get wet?
2. Current estimates show that  $p = 0.6$  in Edinburgh. How many umbrellas should I have so that, if I follow the strategy above, the probability I get wet is less than 0.1?

Smith is in jail and has 3 dollars; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If Smith bets  $A$  dollars, he wins  $A$  dollars with probability 0.4 and loses  $A$  dollars with probability 0.6. Find the probability that he wins 8 dollars before losing all of his money if (a) he bets 1 dollar each time (timid strategy). (b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy). (c) Which strategy gives Smith the better chance of getting out of jail?

In unprofitable times corporations sometimes suspend dividend payments. Suppose that after a dividend has been paid the next one will be paid with probability 0.9, while after a dividend is suspended the next one will be suspended with probability 0.6. In the long run what is the fraction of dividends that will be paid?